CHAPTER SEVEN

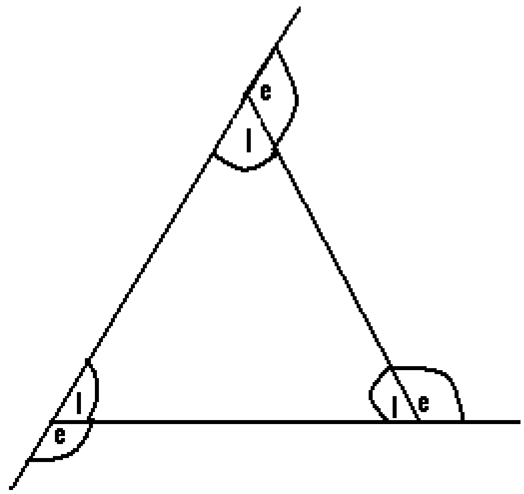
POLYGONS

Definition:

A polygon is a plane figure which is bounded by straight lines.

Polygons	
Number of sides	Name
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon

- A polygon has both interior as well as exterior angles.
- The interior angles of a polygon are those angles which lie within the polygon.
- The exterior angles of a polygon lie outside the polygon.



I = interior angle.

e = exterior angle.

N/B: For any polygon, the sum of the exterior angles = 360^0.

Q1. Calculate the value of each exterior angle of a regular decagon.

<u>Soln.</u>

Decagon has 10 sides and as such 10 exterior angles.

But the sum of the exterior angles of any polygon = 360^0.

$\Rightarrow$ 10 exterior angles = 360^0.

$\therefore$ 1 exterior angle = $\frac{1}{10} \times 360$

$= 36^0$.

$\Rightarrow$ each exterior angle of a decagon = 36^0.

Q2. Find the exterior angle of a regular pentagon.

<u>Soln.</u>

Pentagon has 5 sides, and as such 5 exterior angles. But the sum of the exterior angles of a polygon = 360^0

$\Rightarrow$ 5 exterior angles = 360

$\Rightarrow$ 1 exterior angle = $\frac{1}{5} \times 360$

$= 72^0$.

$\therefore$ Each exterior angle of the regular pentagon $=72^0$. For any polygon, the sum of the exterior angle and the exterior angle at any of its vertices $= 180^0$.

Determination of the interior angle of a regular polygon:

- We must first determine the value of the exterior angle.

- Using the fact that at any vertex, exterior angle + interior angle = 180^0.

$\Rightarrow$ interior angle = 180^0 - exterior angle.

Q1. Calculate the interior angles of a regular decagon.

Soln.

Decagon has 10 exterior angles

$\Rightarrow$ 10 exterior angles = 360^0.

$\therefore$ 1 exterior angle = $\frac{1}{10} \times 360$

$= 36^0$.

But at any vertex, exterior angle + interior angle = 180^0.

$\Rightarrow 36^0$ + interior angle = 180^0.

Interior angle $=180^0 - 36^0 = 144^0$.

The interior angle of the decagon = 144^0.

Q2. Find the value of each Interior angle of a triangle.

Soln.

A triangle has 3 sides and as such 3 exterior angles.

$\Rightarrow$ 3 exterior angles = 360^0

$\therefore$ 1 exterior angle = $\frac{1}{3} \times 360$

$= 120^0.$

But at any vertex, interior angle + exterior angle = 180^0

⇒ Interior angle + 120^0 = 180^0

∴ Interior angle = 60^0

Determination of the sum or the total interior angles of a polygon:

For any polygon, the sum of the interior angles = the number of sides of the polygon × the value of one interior angle.

Q1. Calculate the sum of the interior angles of a regular decagon.

Soln.

Decagon has 10 exterior angles

⇒ 10 exterior angles = 360^0

∴ 1 exterior angle = $\frac{1}{10} \times 360^0$

$= 36^0.$

But at any vertex, interior angle + exterior angle = 180^0

⇒ Interior angle + $36^0 = 180^0$

⇒ Interior angle = 180 – 36

⇒ Interior angle = 144^0.

But the sum of the interior angles of a decagon = interior angle × the number of sides.

∴ Sum of interior angles of the decagon = $144^0 \times 10 = 1440^0$.

Q2. Find the sum of the interior angles of a regular octagon.

Soln.

Octagon has eight sides and as such eight exterior angles.

$\Rightarrow$ 8 exterior angles = 360^0

$\therefore$ 1 exterior angle = $\frac{1}{8} \times 360^0$

= 45^0.

But at any vertex, exterior angle + interior angle = 180^0

$\therefore 45^0$ + interior angle = 180^0

$\Rightarrow$ Interior angle = 180 – 45 = 135^0.

But the sum of interior angle = the number of sides of the polygon $\times$ interior angle = 8 $\times$ 135^0= 1080^0.

Q3.The interior angles of a regular triangle are marked $20^0 + 2x^0, 10^0 + 5x^0$and $40^0 + 4x^0$. Find the actual values of each of these angles.

N/B: First calculate the sum of the interior angles of the triangle.

<u>Soln.</u>

Triangle has 3 exterior angles

$\Rightarrow$ 3 exterior angles = 360^0

$\therefore$ 1 exterior angle = $\frac{1}{3} \times 360^0$

= 120^0.

But at any vertex, exterior angle + interior angle = 180^0

$\Rightarrow$ 120^0 + interior angle = 180^0

$\Rightarrow$ Interior angle = 180^0 - 120^0= 60^0.

But the sum of the interior angles of the triangle = the number of sides $\times$ interior angle = 3 $\times$ 60 = 180^0.

But the interior angles of the triangle are given as $20^0 + 2x^0, 10^0 + 5x^0$ and $40^0 + 4x^0$.The sum of these interior angles = $20^0 + 2x^0 + 10^0 + 5x^0 + 40^0 + 4x^0$

= $20^0 + 10^0 + 40^0 + 2x^0 + 5x^0 + 4x^0 = 70^0 + 11x$.

But the sum of the interior angles of the polygon or triangle = 180^0

$\Rightarrow 70 + 11x = 180^0$

$\Rightarrow 11x = 180^0 - 70 = 110^0$

$\Rightarrow x = \frac{110}{11} = 10^0$.

$\therefore$ The angle marked $20^0 + 2x = 20 + 2(10) = 20^0 + 20^0 = 40^0$.

The angle marked $10^0 + 5x^0 = 10^0 + 50(10) = 10 + 50^0 = 60^0$.

Lastly, the angle marked $40^0 + 4x^0 = 40 + 4(10) = 40 + 40 = 80^0$.